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Aperture Distribution Reconstruction of Phased Array Antenna with Single RF Channel Based on Non-Uniform Spatial Sampling of Array Elements

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***Abstract*—In this paper, a new method is proposed to reconstruct the complex aperture distribution of a phased array antenna with single RF channel. The method employs a non-uniform spatial sampling (NUSS) scheme of array elements. In this scheme, the elements are sequentially sampled with overlapping "on" times so that two or more elements can be simultaneously on. Consequently, this type of sampling passes more average power to the single RF channel in contrast with the uniform sampling which permits only one element to be on while the others are off. To validate the proposed method, it is compared to the time sequence phase weighting (TSPW) technique which uses phase shifters to recover the distribution on the aperture array. It is found that the NUSS system has less computational complexity and consequently it is more suitable for real-time realization. The simulation results also show that it has superior performance over TSPW technique in terms of the error in amplitude and phase of recovered signals. Moreover, the effectiveness of our scheme is demonstrated by analyzing the maximum error in reconstructed aperture distribution with different noise levels and array sizes. Finally, to point out the radiation characteristics of NUSS technique, its array factor (AF) is illustrated and compared to the theoretical one.**

***Index Terms*—phased-arrays, single RF channel, spatial sampling.**

I. INTRODUCTION

The deployment of phased array antennas (smart antennas) for wireless communications has emerged as one of the leading technologies that maximize capacity and improve quality and coverage by dynamically tuning out interference while focusing on the intended user along with impressive advances in the field of digital signal processing [1]. However, there are some hurdles for the deployment of phased-arrays on large scale.

First, Conventional digital beamforming (DBF) is based on the element-space information meaning that an individual complex response from each array element is detected and digitized at the element level. Consequently, each array element is followed by a separate RF channel including low-noise amplifiers (LNAs) and mixers. Therefore, the phased-array system becomes very complex having high power consumption and cost.

On the other hand, single RF channel offers N-fold reduction in hardware, power dissipation, and circuit size for N-element array. Single RF channel receiver is based on beam-space estimation which has less sensitivity to wave front distortion and noise structure and more signal-to-noise ratio (SNR) resolution threshold than the element-space estimation [2].

Many attempts have been made to reduce the number of RF channels to a single RF channel. In [3], antenna array elements are spatially and sequentially sampled and then multiplexed into a single channel. In this scheme, the channel receives $1/N$ of the incoming signal power by the uniform sampling of N array elements. So, the subsequent LNA gain requirement is increased by $10 \log_{10} N$ which represents a problem especially in large arrays.

In [4] and [5], the authors have proposed a DBF antenna array with a single RF channel. In this array, each element is followed by a $0/\pi$ phase shifter to realize the time sequence phase weighting (TSPW) technique in which N different phase weightings are generated to recover each element signal. In this case, the beamformer matrix [6] is ideally Hadamard matrix whose elements are +1 or –1. However, the phase weighting matrix is not ideal in practice due to the imperfection of phase shifters including insertion losses and phase errors. Besides, 180° relative phase shift is only realized in a very narrow bandwidth. As a result, much error is introduced to the reconstructed aperture distribution of the array in both amplitude and phase. In attempt to correct the errors from phase shifters, the authors proposed a correction method in [7] by generating $2N$ weights by the N phase shifters then a matrix formulation is constructed. But the coefficient matrix was proven to be rank deficient and its inverse does not exist, so the pseudoinverse is used instead. In [8], the same authors proposed a method that is simpler and more suitable for real-time correction than the method in [7]. The drawback of the correction method in [8] is that it needs $3N/2$ phase weightings to correct the entire aperture distribution instead of the N weightings generated in the conventional TSPW technique and thus taking more processing and time for correction.

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We introduce in this paper a new approach for the reconstruction of the aperture distribution of the phased array antenna. The approach stems from the non-uniform spatial sampling (NUSS) of array elements and then multiplexed into a single RF channel in the regular fashion. In NUSS technique, the "on" times of individual elements are overlapped so that two or more elements can be simultaneously on. This technique is different from the conventional uniform sampling technique introduced in [3] in which only one element is on while the other elements are off. Consequently, NUSS method passes more power, on average, to the single RF channel. In this way, the gain requirement of LNA of the channel is lessened. Additionally, as a result from not using phase shifters, this method does not suffer from the errors in recovered signal amplitude and phase information emanating from the imperfections of phase shifters. Although the sampling in the proposed method is non-uniform, the sampling rate of all elements is subject to the Nyquist criterion so that the signal amplitude and phase from each antenna element can be restored without loss of fidelity.

In this paper, the operating principle of NUSS technique is first described and then followed by its mathematical foundation. Simulation results are then given with different SNR scenarios in order to validate the performance of the proposed method in different noise environments. The results are compared to those of the correction method in [8]. The comparison shows that the NUSS method leads to maximum amplitude and phase errors in the recovered data that are less than those in [8]. Besides, the variations of maximum error in restored data with SNR and number of array elements are shown to validate the performance of the proposed approach. Finally, the array factor (AF) of NUSS technique is shown and compared to the ideal one to demonstrate the radiation characteristics of the proposed technique.

II. OPERATING PRINCIPLE OF NUSS

The operating principle of NUSS technique is shown in Fig. 1. Only four antenna array elements are taken just for illustrating the principle. First, in State1, only the first element is on via switch S1 and all other elements are off. Then, the switch S2 is activated so that the first two elements are on simultaneously. This is represented by State2. In State3, S3 becomes on making all elements are on except the last element. Finally, in State4, S4 is on and thus all elements become on. This is illustrated in the switch timing diagram in Fig. 2. We can see that the first element is always on via the switch S1. Also, there is overlapping between the "on" intervals of elements causing the average power delivered to the single channel to be more than that of the uniform sampling case. This can be explained as follows. For N-element antenna array, in the beginning, only one element is on so the channel receives $1/N$ of the power of the entire array and then $2/N$ of the incoming power when two elements are on and so forth until all elements are active. At this moment, the channel receives the full power of the array.

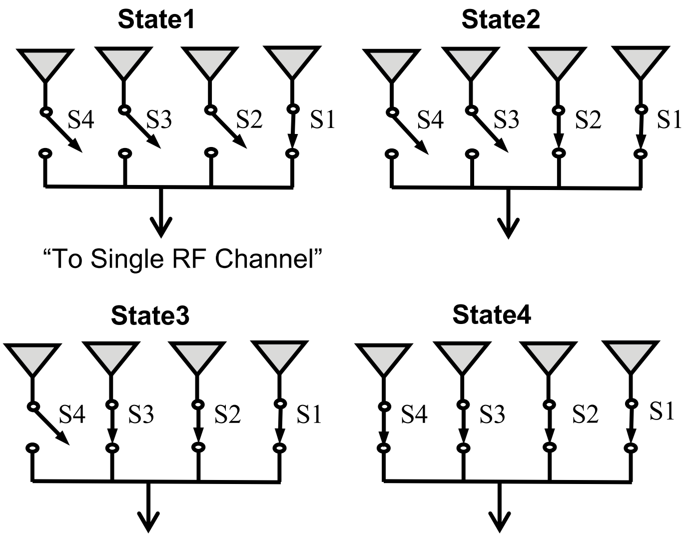

Fig. 1. Operating principle of NUSS technique.

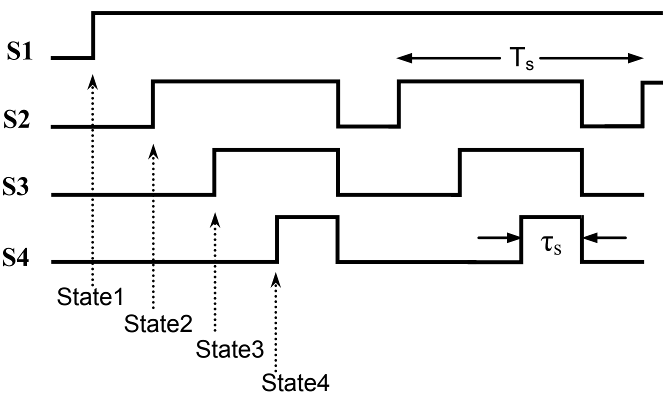

Fig. 2. Switch timing diagram.

As shown in Fig. 2, all elements have the same time period T_s but with different duty cycles. The smallest sampling pulsewidth is of the last element, $\tau_s = T_s / N$. In this case, the switching rate is $f_s = 1/T_s$. To avoid aliasing, each element must be switched at or above the Nyquist rate, so $f_s \geq B \times N$, where N is the number of elements switched in each cycle and B is the signal bandwidth.

III. MATHEMATICAL FOUNDATION

The configuration of NUSS system with N elements is shown in Fig. 3. With the received signal x_k by the kth element, the complex amplitude distribution on the array forms a column vector $\mathbf{X} = [x_1, x_2, \ldots, x_k, \ldots x_N]^{\mathrm{T}}$. If the "on" element is represented by 1 and the "off" element by 0, the switching scheme of NUSS system leads to a unit lower triangular matrix as the beamformer matrix [6], such that the first row of the matrix represents State1 and the second row is state2 and so on until the last row which represents State N. The output of the antenna array is y_k. Therefore, the outputs of the phased array antenna are

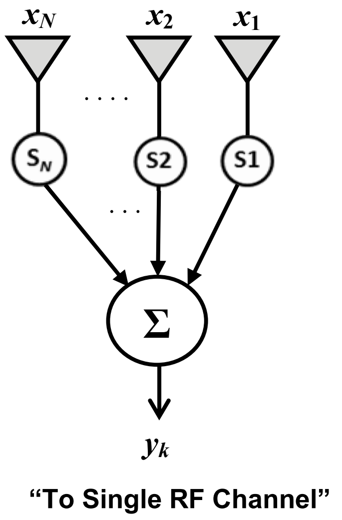

Fig. 3. Configuration of NUSS system with N elements.

$$\begin{bmatrix} y_1 \\ y_2 \\ . \\ . \\ y_N \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & \dots & \dots & 0 \\ 1 & 1 & 0 & \dots & \dots & 0 \\ 1 & 1 & 1 & 0 & \dots & 0 \\ . & . & . & . & \dots & . \\ 1 & 1 & 1 & 1 & \dots & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ . \\ . \\ x_N \end{bmatrix} \tag{1}$$

That is

$$\mathbf{Y} = \mathbf{W}_N \mathbf{X} \tag{2}$$

where $\mathbf{W}_N$ is a unit lower triangular beamformer matrix of order N. The recovery of amplitude and phase distributions on the antenna array aperture is calculated by

$$\mathbf{X} = \mathbf{W}_N^{-1} \mathbf{Y} \tag{3}$$

Where $\mathbf{W}_N^{-1}$ is also a unit lower triangular matrix and is given by

$$\mathbf{W}_N^{-1} = \begin{bmatrix} 1 & 0 & 0 & \dots & \dots & 0 \\ -1 & 1 & 0 & \dots & \dots & 0 \\ 0 & -1 & 1 & 0 & \dots & 0 \\ . & . & . & . & \dots & . \\ 0 & 0 & \dots & 0 & -1 & 1 \end{bmatrix} \tag{4}$$

So, x_k can be easily obtained by the straightforward forward substitution, i.e.,

$$x_1 = y_1, \text{ for } k = 1 \tag{5}$$

and $$x_k = y_k - y_{k-1}, \text{ for } k = 2,\dots, N$$

In this way, The system computational complexity is $O(N)$ in contrast with the Hadamard matrix system used in the TSPW technique [8] which has $O(N^2)$ complexity. Thus, as the size of antenna array N grows, the TSPW technique becomes increasingly dominant in the computational complexity. Additionally, the condition number of the unit lower triangular matrix (or its inverse) is relatively close to 1(it equals 10.65 for $N = 8$) so it is a relatively well-conditioned system.

From the above discussion, it is clear that the NUSS system can be easily implemented in both hardware and software and thus becomes suitable for real-time realization.

IV. SIMULATION RESULTS

The recovery of complex aperture distribution of a single-channel uniform linear array (ULA) antenna with eight elements is simulated using NUSS technique. The space between two adjacent elements is $0.5\lambda_0$, where λ_0 is the wavelength in free space. Assume that there is a plane wave impinging on the array with incident angle $\theta = 3°$. In this case, the elements theoretically have outputs of uniform amplitude and linear progressive phase distribution. As shown in Fig. 4, with no noise, the recovered amplitude and phase distributions conform exactly to the theoretical ones. However, with an additive white Gaussian noise, some errors will be introduced to the recovered distributions as shown for SNRs of 20 and 5 dB.

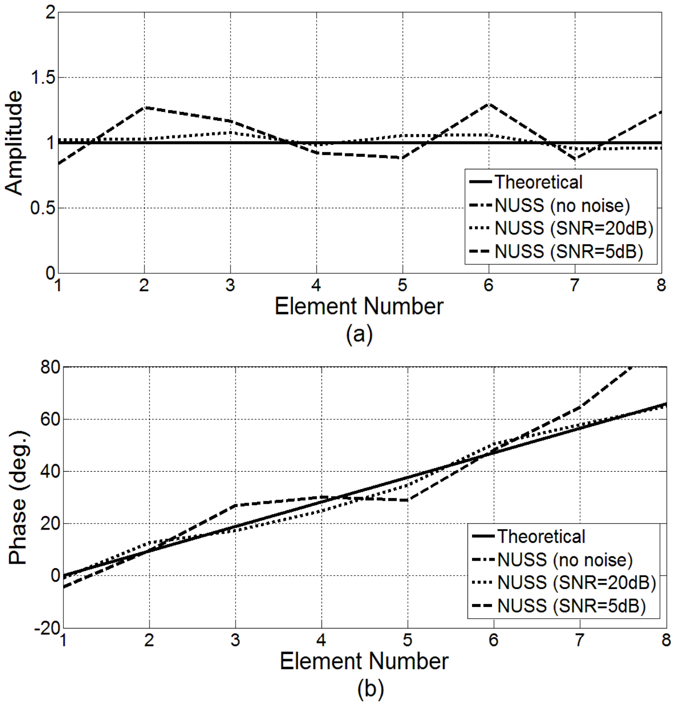

Fig. 4. The theoretical and NUSS aperture distributions with no noise, SNR of 20 dB, and SNR of 5 dB: (a) the amplitude distribution and (b) the phase distribution.

The NUSS method is compared with the method in [8] which is based on the TSPW technique. The comparison is made in terms of the maximum error in recovered amplitude and phase distributions under the same conditions of antenna array configuration, incident angle of plane wave, and noise. The comparison results are summarized in Table I. We can see that the NUSS method has less error in both amplitude and phase distributions than that of the method in [8]. Also, as shown in Table I, the errors increase in [8] when using phase shifters of even worse qualities which is considered as a serious fault in the TSPW technique.

In Fig. 5, the variation of maximum amplitude and phase errors of NUSS technique with SNR is depicted by averaging 1000 iterations. The SNR is ranged from 1 to 20 dB. The incident angle of plane wave is kept constant at $\theta = 3°$. It is obvious that as SNR increases, the maximum error is consistently decreasing from 104.3% for amplitude and from 91.8° for phase.

Fig. 6 shows the variation of maximum error with number of array elements (N). The plane wave incident angle is 3° and SNR is 20 dB. These curves are obtained after ensemble averaging of 10000 iterations. As we can see, the maximum error is increasing logarithmically with N till 20% amplitude error and 11.6° phase error for 128-element ULA. This increasing in error is due to increasing the condition number linearly with increasing N. The solution to this problem can be addressed for large array by dividing it into sub-arrays with each has its own channel instead of using a single channel for the whole array. Also, we can use a non-uniform amplitude distribution for elements (e.g. by using a Chebyshev window) before summing them into the single channel. This may decrease the effect of noise amplification when the number of elements increases.

Finally, Fig. 7 shows the theoretical AF of 8-element ULA, spaced by $0.5\lambda_0$, along with the NUSS AF. As expected, with no noise, the NUSS AF is exactly the same as the theoretical one. However, with added noise, a deviation from theoretical AF begins to appear. This deviation increases with adding more noise as shown in Fig. 7 for two cases of SNR at 20 and 5 dB.

TABLE I. COMPARISON BETWEEEN THE NUSS METHOD AND THE METHOD IN [8]

Maximum Error	**SNR = 20 dB**		
	NUSS	***[8]***	***[8] with worse qualities in phase shifters***
Amplitude	7.7%	9.2%	10.6%
Phase	3.4°	6.3°	7.5°
	SNR = 5 dB		
Amplitude	29.8%	About 40%	
Phase	24.6°	About 30°	

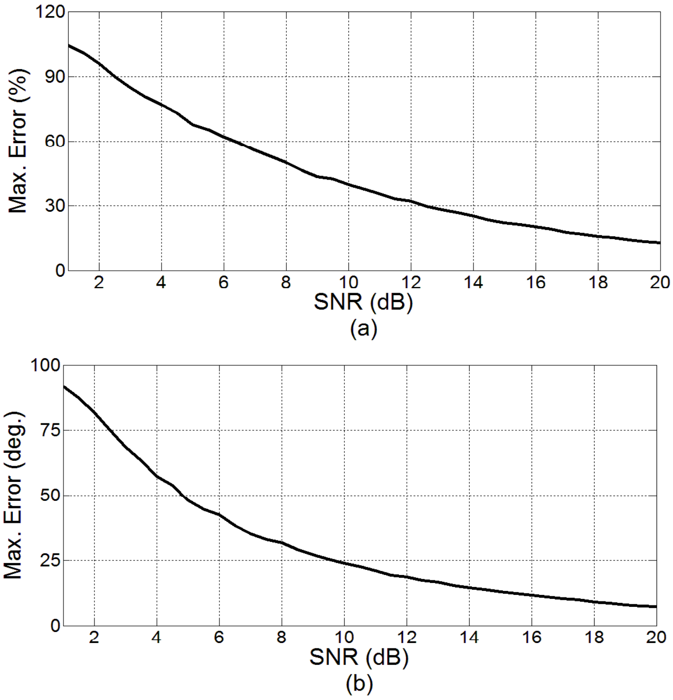

Fig. 5. Maximum error in recovered distributions of NUSS versus SNR: (a) amplitude and (b) phase.

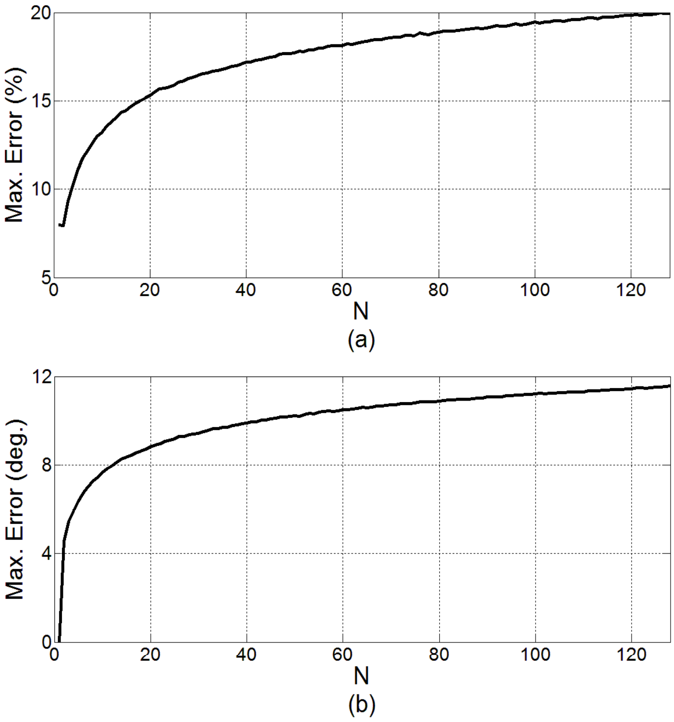

Fig. 6. Maximum error versus number of array elements: (a) amplitude and (b) phase.

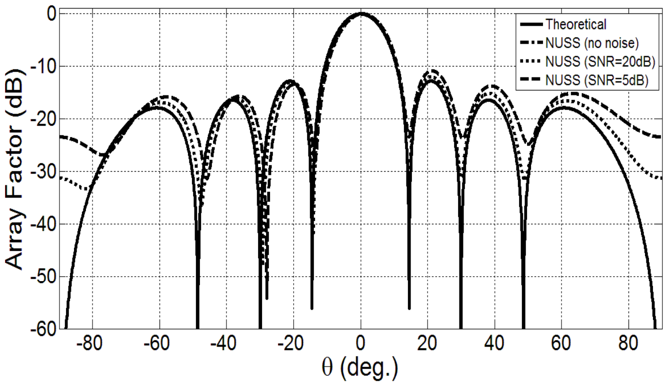

Fig. 7. The theoretical and NUSS array factors.

V. CONCLUSION

The reconstruction of the aperture distribution of the phased array antenna with single RF channel using a new NUSS technique has been proposed. The new technique utilizes the non-uniform sampling of the array elements and then multiplexed into the single channel. In contrast with the uniform sampling technique, the new method allows more average power to be delivered to the single channel and thus alleviating the gain requirement of channel's LNA. Compared to the TSPW technique, the simulation results show that the NUSS method has less error in the recovered amplitude and phase distributions. Also, the triangular system of NUSS technique has shown to have less computational complexity than the Hadamard system of TSPW technique. In other words, the NUSS method is more suitable for the real-time recovery of the phased array antenna aperture distribution. Moreover, in NUSS technique, it was shown that as SNR increases, the maximum error monotonically decreases. Also, the NUSS method has shown to have a logarithmic increase in maximum error as the number of array elements increases due to the increase in condition number. This can be addressed for large arrays by using sub-single-channels and non-uniform amplitude distribution for elements. Finally, it has been shown that the NUSS technique has an AF that is in a good agreement with the ideal one even at 5 dB SNR. Once the amplitude and phase information available at each array element have been recovered, direction of arrival estimation algorithms and DBF can be applied—further study.

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